## Strategic competition

American term: Industrial organization
A better name: The economics of industry

- the study of activities within an industry, mainly with respect to competition among the firms in a product market.

Why is this topic important?

- The model of perfect competition is unrealistic.
- Who set the prices?
- The firms.
- Can they influence the price?
- Yes, for example if their products differ, or if they are few.

But: difficult to find a general model of imperfect competition.
$\rightarrow$ Many models with varying applications

- Is it smart to have a whole battery of models?
- The predictions from the perfect-competition model do not fit. In many industries:
- high profits
- $p>M C$


## The study of an industry

- few firms
- partial equilibrium
- how do the firms compete with each other?
- setting prices? quantities?
- making investments? advertising? R\&D? capacity?
- location of outlets
- what do they do to avoid competition?
- product differentiation
- entry deterrence
- predatory actions
- collusion
- merger

Various models, all with the same analytical tool: game theory

## What is the right model to use?

- What kind of market are we looking at?

Example: market for petrol vs. market for cars
petrol: homogeneous good
car: heterogeneous good
petrol: easy for firms to supervise each other's prices car: price supervision difficult

Product differentiation $\rightarrow$ weaker competition
$\rightarrow$ petrol market more competitive
Price supervision: easy to coordinate on prices
$\rightarrow$ petrol market less competitive
Both markets may have the same mark-up, but explanations may differ.

In order to understand how firms in an industry compete (or not), we need a catalogue of different models.

Even in the study of a single industry, it may be helpful to have different models of strategic competition in mind.

Example: Norwegian airlines.


## - Predation

- Entry deterrence
- Non-price competition
- Collusion
- Consumer switching costs


## Central concepts from game theory

- Extensive form vs. normal form
- Strategy vs. action
- Pure strategy vs. mixed strategy
- Dominated strategy
- Nash equilibrium
- Subgame-perfect equilibrium
- Repeated games

Repetition of game theory:

Tirole, secs 11.1-11.3 (for ch 9: secs 11.4-11.5)
Exercises 11.1, 11.4, 11.9.

## Competition in the short run

or: Static oligopoly theory
Firms make decisions simultaneously
Actions chosen from continuous action spaces
Differentiable profit functions
First-order conditions

Nash equilibrium with 2 firms:

$$
\frac{\partial \pi^{i}\left(s_{1}^{*}, s_{2}^{*}\right)}{\partial s_{i}}=0 \quad i=1,2
$$

Each firm's decision is optimum, given the other firm's equilibrium decision.

The other firm's decision is exogenous.
Thus, we can find one firm's optimum decision given the other firm's choice: Best-response functions
$R_{1}\left(s_{2}\right)$ is firm 1's best-response function, defined by:

$$
\frac{\partial \pi^{1}\left(R_{1}\left(s_{2}\right), s_{2}\right)}{\partial s_{1}}=0
$$

Best-response curves:


The slope of the best-response curve:

$$
\begin{aligned}
& \frac{\partial^{2} \pi^{1}}{\partial s_{1}^{2}} d R_{1}+\frac{\partial^{2} \pi^{1}}{\partial s_{1} \partial s_{2}} d s_{2}=0 \\
& \Rightarrow R_{1}^{\prime}\left(s_{2}\right)=\frac{d R_{1}}{d s_{2}}=-\frac{\frac{\partial^{2} \pi^{1}}{\partial s_{1} \partial s_{2}}}{\frac{\partial^{2} \pi^{1}}{\partial s_{1}^{2}}}
\end{aligned}
$$

Second-order condition $\Rightarrow \frac{\partial^{2} \pi^{1}}{\partial s_{1}^{2}}<0$
Therefore: $\operatorname{sign} R_{1}{ }^{\prime}\left(s_{2}\right)=\operatorname{sign} \frac{\partial^{2} \pi^{1}}{\partial s_{1} \partial s_{2}}$
$\frac{\partial^{2} \pi^{1}}{\partial s_{1} \partial s_{2}}<0:$
An increase in $s_{2}$ implies a reduction in firm 1's payoff from a marginal increase in $s_{1}$. This implies a reduction in firm 1's optimum. The two firms' choice variables are strategic substitutes.

$$
\frac{\partial^{2} \pi^{1}}{\partial s_{1} \partial s_{2}}>0
$$

An increase in $s_{2}$ implies an increase in firm 1's payoff from a marginal increase in $s_{1}$. This implies an increase in firm 1's optimum. The two firms' choice variables are strategic complements.

Generally, but not always:

- prices are strategic complements
- quantities are strategic substitutes


## Price competition

A firm's price is a short-term commitment. So a regular picture of competition in the short run is one of competition in prices.

Modelling is a trade-off between making a model

- simple, so that we can understand it; and
- reasonable, so that we can use it.

Let us start out with simplicity.
Two firms, homogeneous goods (perfect substitutes). Consumers care only about price.
Market demand: $D(p), D^{\prime}<0$.
Constant unit cost: $c$.
No capacity constraints.
Firms choose prices simultaneously and independently.
Equilibrium prices - Bertrand equilibrium.
(Joseph Bertrand, 1883)
Firm 1's profit:

$$
\begin{array}{r}
\pi^{1}\left(p_{1}, p_{2}\right)=\left(p_{1}-c\right) D_{1}\left(p_{1}, p_{2}\right), \text { where } \\
D_{1}\left(p_{1}, p_{2}\right)= \begin{cases}D\left(p_{1}\right), & \text { if } p_{1}<p_{2} \\
\frac{1}{2} D\left(p_{1}\right), & \text { if } p_{1}=p_{2} \\
0, & \text { if } p_{1}>p_{2}\end{cases}
\end{array}
$$

$\pi^{1}\left(p_{1}, p_{2}\right)$ is discontinuous, because $D_{1}\left(p_{1}, p_{2}\right)$ is.
First-order approach not applicable.
Nash equilibrium:

$$
\begin{aligned}
& \pi^{1}\left(p_{1}{ }^{*}, p_{2}{ }^{*}\right) \geq \pi^{1}\left(p_{1}, p_{2}{ }^{*}\right), \forall p_{1} . \\
& \pi^{2}\left(p_{1}{ }^{*}, p_{2}{ }^{*}\right) \geq \pi^{2}\left(p_{1}{ }^{*}, p_{2}\right), \forall p_{2} .
\end{aligned}
$$

Result: There exists a unique equilibrium, in which

$$
p_{1}{ }^{*}=p_{2}^{*}=c
$$

Two steps in the proof.
Step 1: This is an equilibrium.
Step 2: No other price combination is an equilibrium.

[Exercise 5.1: cost asymmetry]

The same result holds for any number of firms $\geq 2$. So there is nothing between monopoly and perfect competition (the Chicago school).
Or is there?
The model lacks realism.
Resolving the Bertrand paradox
(i) Product differentiation

Consumers care for both price and product characteristics.

No longer true that $R(c)=c$.
If $p_{2}=c$, then $p_{1}+\varepsilon$ provides firm 1 with positive profit.
Thus, $p=c$ no longer equilibrium.
[Lectures 4 and 5]
(ii) Time horizon

Consider the case $p_{1}=p_{2}>c$. Not an equilibrium, because firm 1 is better off with reducing its price strictly below $p_{2}$. But what if firm 2 can respond to this? Would it set a price even lower? If so, could it be that firm 1 does not have incentives for a price reduction to start with?
[Lectures 2 and 3]
(iii) Capacity constraints

Firms cannot sell more than they are able to produce.

Capacity constraints: $\bar{q}_{1}$ and $\bar{q}_{2}$.

Suppose $\bar{q}_{1}<D(c)$.
$p=c$ is no longer equilibrium
Suppose firm 1's price is $p_{1}=c$. If now firm 2 sets $p_{2}=c$ $+\varepsilon$, then firm 1 faces a higher demand than its capacity. Some consumers will have to go to the high-price firm 2, who therefore earns a profit.

Capacity constraints are an extreme version of decreasing returns to scale.


[This lecture]

## Price competition with capacity constraints

Consumers are rationed at the low-price firm. But who are the rationed ones?

As before: two firms; homogeneous goods.

## Efficient rationing

If $p_{1}<p_{2}$ and $\bar{q}_{1}<D\left(p_{1}\right)$, then the residual demand facing firm 2 is:

$$
\widetilde{D}_{2}\left(p_{2}\right)= \begin{cases}D\left(p_{2}\right)-\bar{q}_{1}, & \text { if } D\left(p_{2}\right)>\bar{q}_{1} \\ 0, & \text { otherwise }\end{cases}
$$



This is the rationing that maximizes consumer surplus: The consumers with the highest willingness to pay get the low price.

## Proportional rationing

Let $p_{1}<p_{2}$ and $\bar{q}_{1}<D\left(p_{1}\right)$.

Instead of favouring the consumers with the highest willingness to pay, all consumers have the same chance of getting the low price.

Probability of being supplied by the low-price firm 1 is:

$$
\frac{\bar{q}_{1}}{D\left(p_{1}\right)}
$$

The residual demand facing the high-price firm 2 is:

$$
\widetilde{D}_{2}\left(p_{2}\right)=D\left(p_{2}\right)\left[1-\frac{\bar{q}_{1}}{D\left(p_{1}\right)}\right]
$$



Not efficient - some consumers get supplies despite having a willingness to pay below $p_{2}$, consumers' marginal cost.

## Example

Two firms, homogeneous demand: $D(p)=1-p$

Zero marginal costs of production: $c=0$.

High investment costs have led to low capacity:
$\bar{q}_{1}=\bar{q}_{2} \leq \frac{1}{3}$.

Assume efficient rationing.
Define: $p^{*}=1-\left(\bar{q}_{1}+\bar{q}_{2}\right)$. [Note: $p^{*} \geq \frac{1}{3}>c$.]

Is $p_{1}=p_{2}=p^{*}$ an equilibrium?
Note that $D\left(p^{*}\right)=\bar{q}_{1}+\bar{q}_{2}$; total capacity exactly covers demand at this price.

Can another price be preferable for firm 1 to $p^{*}$, if firm 2 sets $p_{2}=p^{*}$ ?
(i) Consider $p_{1}<p_{2}=p^{*}$. A lower price for firm 1 without any increase in sales.
(ii) Consider $p_{1}>p_{2}=p^{*}$. Firm 1's sales less than before:

$$
\begin{aligned}
& q_{1}=\widetilde{D}_{1}\left(p_{1}\right)=D\left(p_{1}\right)-\bar{q}_{2}=1-p_{1}-\bar{q}_{2} \\
& \Rightarrow p_{1}=1-q_{1}-\bar{q}_{2}
\end{aligned}
$$

## Profit of firm 1:

$$
\pi_{1}=p_{1} \widetilde{D}_{1}\left(p_{1}\right)
$$

Equivalently:

$$
\pi_{1}=\left(1-q_{1}-\bar{q}_{2}\right) q_{1}
$$

Is it profitable for firm 1 with a price above $p^{*}$ ?
Equivalently: Is it profitable with a quantity below $\bar{q}_{1}$ ?

$$
\frac{d \pi_{1}}{d q_{1}}=1-2 q_{1}-\bar{q}_{2}
$$

Second-order condition: $\partial^{2} \pi_{1} / \partial q_{1}{ }^{2}<0$.

$$
\left.\frac{d \pi_{1}}{d q_{1}}\right|_{q_{1}=\bar{q}_{1}}=1-2 \bar{q}_{1}-\bar{q}_{2} \geq 0
$$

Optimum is at $\bar{q}_{1}$.
Thus, the optimum price for firm 1 is $p^{*}$. Equivalently for firm 2. Thus, $p_{1}=p_{2}=p^{*}$ in equilibrium.

Is this equilibrium unique? Yes.
Larger capacities: No equilibria in pure strategies.
[Exercise 5.2]

## Capacity a more long-term decision than price?

Consider the two-stage game:
Stage 1: Firms choose capacities
Stage 2: Firms choose prices
Investment costs: $c_{0}$ per unit of capacity
Suppose $c_{0}$ is so high that, in equilibrium, capacities will be low. We can then make use of our analysis of the price game: Prices equal $p^{*}$.

Profit net of investment costs:

$$
\pi^{1}\left(\bar{q}_{1}, \bar{q}_{2}\right)=\left\{\left[1-\left(\bar{q}_{1}+\bar{q}_{2}\right)\right]-c_{0}\right\} \bar{q}_{1} .
$$

Now, the game is equivalent to a one-stage game in capacities where demand $=$ total capacity $=$ total supply.

That is, a one-stage game in quantities. (Cournot, 1838)
With efficient rationing and a concave demand function, the two games are equivalent in equilibrium outcome, for all $c_{0}$.

Therefore, a model of one-stage quantity competition, with prices coming from nowhere, can be understood as a simple substitute for a more realistic but more complex model where firms compete in capacities and thereafter in prices.

## The Cournot model

Two firms choose quantities simultaneously.
Costs: $C_{i}\left(q_{i}\right)$
Total production: $Q=q_{1}+q_{2}$
Inverse demand: $P(Q), \quad P^{\prime}<0$.
Profit, firm 1:

$$
\pi^{1}\left(q_{1}, q_{2}\right)=q_{1} P\left(q_{1}+q_{2}\right)-C_{1}\left(q_{1}\right)
$$

First-order condition:

$$
\frac{\partial \pi^{1}}{\partial q_{1}}=P\left(q_{1}+q_{2}\right)+q_{1} P^{\prime}\left(q_{1}+q_{2}\right)-C_{1}^{\prime}\left(q_{1}\right)=0
$$

$q_{1} P^{\prime}\left(q_{1}+q_{2}\right) \quad-\quad$ the infra-marginal effect of an increase in quantity

Equilibrium: $\frac{\partial \pi^{1}}{\partial q_{1}}=0 ; \frac{\partial \pi^{2}}{\partial q_{2}}=0$.

For firm 1:

$$
\begin{aligned}
& P-C_{1}^{\prime}=-q_{1} P^{\prime}=-\frac{q_{1}}{Q} P^{\prime} Q=-\frac{\frac{q_{1}}{Q}}{\frac{1}{P^{\prime}} \frac{1}{Q}} \\
& \Rightarrow \frac{P-C_{1}^{\prime}}{P}=\frac{\frac{q_{1}}{Q}}{-\frac{1}{P^{\prime}} \frac{P}{Q}}
\end{aligned}
$$

$L_{1}=\frac{P-C_{1}^{\prime}}{P}-\quad$ the Lerner index of firm 1

$$
\alpha_{1}=\frac{q_{1}}{Q} \quad-\quad \text { firm 1's market share }
$$

$D(p) \quad-\quad$ the market demand

$$
\begin{aligned}
& D(P(Q)) \equiv Q \\
& \Rightarrow D^{\prime}(p) P^{\prime}(Q)=1
\end{aligned}
$$

Demand elasticity:
$\varepsilon=-D^{\prime} \frac{P}{D}=-\frac{1}{P^{\prime}} \frac{P}{Q}$
$\Rightarrow L_{1}=\frac{\alpha_{1}}{\varepsilon}$
Note:
(i) $\alpha_{1} / \varepsilon>0 \Rightarrow L_{1}>0 \Rightarrow P>C_{1}{ }^{\prime}$.
(ii) Monopoly: $\alpha_{1}=1$, and $L_{1}=1 / \varepsilon$.
$n$ firms: $Q=\sum_{i=1}^{n} q_{i}$

$$
\begin{aligned}
& \pi^{i}\left(q_{1}, \ldots, q_{n}\right)=q_{i} P(Q)-C_{i}\left(q_{i}\right) \\
& \frac{\partial \pi^{i}}{\partial q_{i}}=P(Q)+q_{i} P^{\prime} \underbrace{\frac{d Q}{d q_{i}}}_{=1}-C_{i}^{\prime}=0
\end{aligned}
$$

Example: $P(Q)=a-Q$;

$$
C_{i}\left(q_{i}\right)=C\left(q_{i}\right)=c q_{i} \text {, where } a>c .
$$

First-order condition firm $i: a-Q-q_{i}-c=0$.
All firms identical $\Rightarrow q_{1}=\ldots=q_{n}=q, Q=n q$
Applied to the first-order condition:

$$
\begin{aligned}
& a-n q-q-c=0 \\
& q=\frac{a-c}{n+1} \\
& P=a-n q=a-\frac{n(a-c)}{n+1}=\frac{a+n c}{n+1}=c+\frac{a-c}{n+1}>c \\
& Q=n q=\frac{n}{n+1}(a-c) \\
& \pi=q\left(c+\frac{a-c}{n+1}\right)-c q=\frac{a-c}{n+1} q=\left(\frac{a-c}{n+1}\right)^{2} \\
& n \rightarrow \infty \Rightarrow P \rightarrow c, Q \rightarrow a-c, \pi \rightarrow 0 .
\end{aligned}
$$

[Exercises 5.3, 5.4, 5.5]

## Bertrand vs. Cournot

Competing models? - No.
Firms set prices.
When capacity constraints are of little importance, the Bertrand model is the preferred one.
When capacity constraints are present to an important extent (decreasing returns to scale), the Cournot model is the best choice.

## Measuring concentration

A substitute for measuring price-cost margins, since costs are unobservable.

A popular measure: the Herfindahl index.

$$
R_{H}=\sum_{i=1}^{n} \alpha_{i}^{2}
$$

Model: $n$ firms, $C_{i}\left(q_{i}\right)=c_{i} q_{i}$, quantity competition
Total industry profits:
$\sum_{i} \pi_{i}=\sum_{i}\left(P-c_{i}\right) q_{i}=\sum_{i} \frac{P \alpha_{i} q_{i}}{\varepsilon}=\frac{P Q}{\varepsilon} \sum_{i} \alpha_{i}^{2}=\frac{D^{2}}{-D^{\prime}} R_{H}$
Assume: $\varepsilon=1 \Rightarrow p D(p)=k \Rightarrow D(p)=k / p$
$\Rightarrow D^{2} /\left(-D^{\prime}\right)=k \Rightarrow \sum_{i} \pi_{i}=k R_{H}$
The Herfindahl index is proportional to total industry profits.
[Exercises 5.6, 5.7]

